

# Suppression of stochastic fluctuations of suspended nanowires by temperature-induced single-electron tunnelling

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**Abstract.** We investigate theoretically the electromechanical properties of freely suspended nanowires that are in tunnelling contact with the tip of a scanning tunnelling microscope (STM) and two supporting metallic leads. The aim of our analysis is to characterize the fluctuations of the dynamical variables of the nanowire when a temperature drop is maintained between the STM tip and the leads, which are all assumed to be electrically grounded. By solving a quantum master equation that describes the coupled dynamics of electronic and mechanical degrees of freedom we find that the stationary state of the mechanical oscillator has a Gaussian character, but that the amplitude of its root-mean square center-of-mass fluctuations is smaller than would be expected if the system were coupled only to the leads at thermal equilibrium.

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## 1. Introduction

The possibility to detect and control the motion of nanometer-sized-mechanical resonators by coupling them to mesoscopic electronic devices has generated a considerable research effort in recent years [1]. In particular, the possibility to use such nanoelectromechanical systems (NEMS) as ultrasensitive sensors of, for example, displacement and mass, have been demonstrated in a number of works [2, 3]. Independently of the specific type of electronic device considered in the different schemes, a common feature that has emerged from these studies is that the electronic subsystem must be out of thermodynamic equilibrium in order to function as an ultrasensitive measurement tool in combination with the mechanical subsystem. This observation naturally raises the question of how the dynamics of the mechanical subsystem is affected by the *nonequilibrium environment* created by the mesoscopic electronic device to which it is coupled.

It is known from statistical mechanics that the displacement and momentum fluctuations of a quantum harmonic oscillator (the basic model for any movable structure that could be included in a NEMS), which is coupled to a thermal bath in equilibrium at temperature  $T$  are described by Gaussian distribution functions, whose widths are proportional to  $\coth^{1/2}(\hbar\omega/2k_B T)$  where  $\omega$  is the frequency of the oscillator. From this formula it follows that the fluctuations have a thermodynamic origin in the high-temperature limit, where they are fully defined by the temperature of the thermal bath. On the other hand, at low temperatures  $k_B T \ll \hbar\omega$  the fluctuations are completely quantum mechanical in nature.

What kind of changes from this picture could be expected if the oscillator is coupled to a *nonequilibrium* environment? In spite of the difficulties related to the definition of temperature for systems that are out of thermodynamic equilibrium, several theoretical works show that nonequilibrium fluctuations in the properties of nanomechanical oscillators coupled to mesoscopic electronic systems (such as a single-electron transistor or a superconducting Cooper pair box) are, to a good approximation, still described by Gaussian distribution functions [4, 5].

A remarkable difference between a passive (thermodynamic) environment and an active (nonequilibrium) one is that in the latter case the amplitude of the fluctuations can be controlled through parameters that characterize the state of the electronic subsystem. This external control introduced by the coupling between mechanical and electronic degrees of freedom opens the way for the possibility to reach the quantum limit of fluctuations even when the temperature is high on the scale defined by the quantum of mechanical energy, i.e.  $\hbar\omega/k_B$ .

In the last few years considerable efforts have been made in order to, develop efficient procedures to effectively “cool” down the motion of nanomechanical resonators below the threshold defined by thermal fluctuations. Most of the proposed schemes strive to reproduce the effects of laser cooling of atoms and molecules by purely electronic means [6, 8, 7, 9]. The general strategy underlying these approaches is based on the coherent

control of resonant, energy-conserving transitions between *discrete* electronic levels.

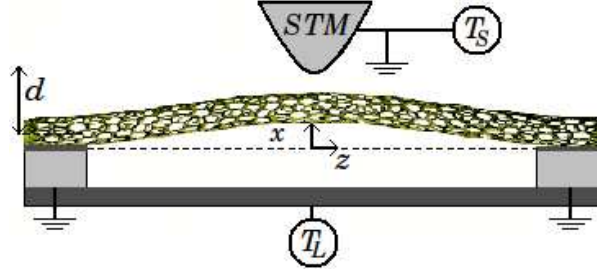
Recently we have proposed a fundamentally new scheme to cooling the vibrations of a suspended-nanowire based mechanical oscillator [10]. In contrast to the aforementioned cooling schemes, our proposal has the advantage that it does not rely on the energy conservation constraint. In particular, we considered a suspended carbon nanotube in tunnelling contact with the voltage-biased tip of a scanning tunnelling microscope (STM) and the metallic leads at which its ends are clamped. Our analysis showed that the average number of quantized vibrational excitations, i.e. vibrons, (which is proportional to the root-mean square fluctuations of the center-of-mass position) can be reduced by varying the bias voltage within a range of values for which the probability for absorbing vibrons during inelastic electron tunnelling processes is significantly enhanced over the probability for vibron emission.

In this paper we demonstrate that the cooling mechanism suggested in Ref. 10 can work by exploiting the temperature gradient, rather than the potential drop across the system. We show that the amplitude of the root-mean square fluctuations of the nanotube center-of-mass position is smaller than what would be in presence of only an equilibrium thermal bath. This partial suppression of the stochastic fluctuations of the nanotube displacements can be interpreted as an effective cooling of the mechanical degrees of freedom of the system. Moreover, we have found that this effective cooling phenomenon involves simultaneously several low-frequency vibrational modes and not only the fundamental one.

## 2. Model Hamiltonian

To be specific, we consider the system sketched in Fig. 1, where a carbon nanotube is suspended over a trench between two metallic leads. Its ends are both clamped, while a third electrode in the form of the tip a scanning tunnelling microscope (STM) is positioned above the nanotube. The suspended carbon nanotube can be considered as a quantum dot that is coupled to the surrounding electrodes through tunnel junctions. Low-temperature tunnelling spectroscopy studies on freely hanging carbon nanotubes have shown that inelastic electron tunnelling can create a non-thermal equilibrium population of the vibronic states of the nanotube [11].

In order to analyze the dynamics of the nanotube deflections and the behavior of the electronic subsystem in the quantum regime we introduce a model Hamiltonian,



**Figure 1.** Sketch of the model system considered. A carbon nanotube is suspended over a trench between metallic leads, while an STM tip is placed a distance  $d$  above the nanotube. Both the STM and the leads are grounded, while their temperatures  $T_S$  and  $T_L$  are different and held constant ( $T_S > T_L$  is assumed).

$H = H_e + H_m + H_T + H_C$ , where the different contributions are given by:

$$H_e = \sum_{q,\alpha} E_{q,\alpha} a_{q,\alpha}^\dagger a_{q,\alpha} + E_0 c^\dagger c, \quad (1a)$$

$$H_m = \int_{-L/2}^{L/2} dz \left\{ \frac{\hat{\pi}^2(z)}{2\rho} + \frac{\kappa [\hat{u}''(z)]^2}{2} \right\}, \quad (1b)$$

$$H_T = \sum_q \left\{ t_S [\hat{u}(z_0)] c^\dagger a_{q,S} + t_L a_{q,L}^\dagger c \right\} + \text{H.c.}, \quad (1c)$$

$$H_C = -\mathfrak{S} \hat{u}(z_0) c^\dagger c. \quad (1d)$$

In Eqs. (1a), (1c) and (1d),  $a_{q,\alpha}^{(\dagger)}$  and  $c^{(\dagger)}$  are annihilation (creation) operators for electrons in the STM tip ( $\alpha = S$ ), in the leads ( $\alpha = L$ ) and in the nanotube, respectively. The term  $H_e$  in Eq. (1a) describes the electronic states in the STM tip, the leads (which are treated as reservoirs of non-interacting quasiparticles) and in the nanotube. We assume that, in the range of temperatures that are relevant for our considerations, only one quantized electronic level in the nanotube is involved in the exchange of charge with the reservoirs and we denote its energy as  $E_0$ . Such a condition is satisfied if the temperatures of the reservoirs are significantly lower than the characteristic difference in energy between the quantized electronic levels of the nanotube, which can be estimated as  $\Delta \simeq \hbar v_F / 2\ell \approx 1.7 \text{ meV}/\mu\text{m}$ , where  $v_F$  is Fermi velocity and  $\ell$  is the length of the nanotube.

The term  $H_m$  in Eq. (1b) describes the mechanical degrees of freedom of the nanotube. The quantum field  $\hat{u}(z)$  gives the nanotube deflection from the straight configuration at point  $z$  (that is, the nanotube axis, see Fig. 1), while  $\hat{\pi}(z)$  is the momentum linear density and the symbol  $'$  denotes derivation with respect to the coordinate  $z$ . The displacement and momentum density fields are canonically conjugated dynamical variables, that is they obey the commutation relation:  $[\hat{u}(z_i), \hat{\pi}(z_j)] = i\hbar\delta(z_i - z_j)$ .

The parameter  $\rho$  represents the linear mass density of the nanotube,  $\kappa$  its bending rigidity, and  $L$  is the length of the suspended part (notice that  $L \neq \ell$ , where the latter

is the total length of the nanotube). The clamping of both the ends of the nanotube to the leads can be expressed through the boundary conditions  $\hat{u}(\pm L/2) = \hat{u}'(\pm L/2) = 0$ .

The tunnelling of electrons through the STM tip-nanotube and the nanotube-leads junctions is described by the Hamiltonian operator  $H_T$ , presented in Eq. (1c). We denote by  $z_0$  the point along the nanotube axis above which the STM is positioned. Both the tunnelling amplitudes  $t_S[\hat{u}(z_0)]$  and  $t_L$  are assumed to be independent of the electronic energy, whereas only  $t_S[\hat{u}(z_0)]$  is a function of the nanotube deflection, as a consequence of its dependence on the overlap of the electronic wavefunctions in the STM tip and the nanotube. We model this deflection dependence of the probability amplitude of tunnelling between the STM tip and the nanotube as  $t_S[\hat{u}(z_0)] \equiv t_S \exp[u(z_0)/\lambda]$ , where  $\lambda$  is the characteristic tunnelling length of the junction ( $\lambda \sim 10^{-10}$  m).

The effect that the nanotube displacement has on the width of the tunnel barrier provides a mechanism of coupling the electronic and the mechanical degrees of freedom of the system. In the following, we will refer to that as *tunnelling* electromechanical coupling.

The last term in the Hamiltonian,  $H_C$ , shown in Eq. (1d), describes the electrostatic interaction in the system. Since both the STM tip and the substrate are grounded, an electron occupying the state inside nanotube induces a polarization charge of opposite sign in the STM tip and hence generates an electrostatic force  $\mathfrak{F}$  acting on the nanotube. This force can be thought as applied to the point  $z_0$  and directed towards STM tip (this “strongly localized” form of the electrostatic force is a valid approximation if the effective radius of the STM tip is negligible with respect to the length of the nanotube).

The electrostatic interaction described by the operator  $H_C$  provides another mechanism that couples the dynamics of the mechanical and electronic degrees of freedom, which we will refer to hereafter as the *polaronic* electromechanical coupling, because of the formal analogy with the interaction term in Hamiltonian of the polaron problem. Under the assumptions of uniform charge distribution inside the charged nanotube, if the equilibrium distance between the nanotube and the STM tip  $d$  is much less than the effective radius of the tip,  $R$ , then  $\mathfrak{F}$  can be approximated by the following expression:

$$\mathfrak{F} = \frac{\beta}{\varepsilon_0} \left( \frac{R}{\ell^2} \right) \frac{e^2}{d}, \quad (2)$$

where  $\beta$  is a numerical factor of the order of one and  $\varepsilon_0$  the vacuum permittivity. From the considerations presented above, it follows that the electronic and mechanical subsystems interact through two independent coupling mechanisms, that is the tunnelling and the polaronic. The former one results in the change of the nanotube momentum on the value  $\hbar/\lambda$  when one electron tunnels from the STM to the nanotube or in the opposite direction. The second one accounts the difference between the equilibrium configurations of the charged and neutral nanotube. Working incoherently these two mechanisms of electromechanical coupling give rise to stochastic fluctuations of the nanotube center-of-mass position.

However, as it was shown in Ref. 10, the quantum interplay between them may

significantly modify their mutual performance. In order to analyze the consequences of the combination of the two coupling mechanisms, it is convenient to introduce the eigenmode representation for the nanotube displacement and momentum density fields, that is defined by the operators

$$\hat{X}_n = \frac{1}{\sqrt{L}} \int_{-L/2}^{L/2} dz \varphi_n(z) \hat{u}(z) \quad (3a)$$

$$\hat{P}_n = \sqrt{L} \int_{-L/2}^{L/2} dz \varphi_n(z) \hat{\pi}(z). \quad (3b)$$

The operators presented in Eqs. (3a) and (3b) satisfy canonical commutation relations, that is  $[\hat{X}_n, \hat{P}_l] = i\hbar\delta_{n,l}$  and the complete set of orthonormal functions  $\varphi_n(z)$  is given by the eigenfunctions of the operator  $d^4/dz^4$  with the boundary conditions  $\varphi_n(\pm L/2) = \varphi'_n(\pm L/2)$ . In this representation the terms  $H_m, H_T$  and  $H_C$  in the Hamiltonian assumes the form:

$$H_m + H_C = \sum_n \left( \frac{1}{2M} \hat{P}_n^2 + \frac{\omega_n^2 M}{2} \hat{X}_n^2 \right) - \Im c^\dagger c \sum_n \varphi_n(z_0) \hat{X}_n, \quad (4)$$

$$H_T = \sum_q \left[ t_S e^{\sum_n \varphi_n(z_0) \hat{X}_n / \lambda} c^\dagger a_{q,S} + t_L a_{q,L}^\dagger c \right] + \text{H.c.}, \quad (5)$$

where  $M = \rho L$  is the mass of the suspended part and  $\omega_n$  is the eigenfrequency of the  $n$ -th bending mode. The polaronic term can be removed from the Hamiltonian by a suitable unitary transformation,  $H \rightarrow \tilde{H} = U H U^\dagger$ , where  $U \equiv \exp[i\hbar^{-1} \Im c^\dagger c \sum_n \varphi_n(z_0) \hat{P}_n / 2M\omega_n^2]$ . However, as additional consequence of this transformation, the tunnelling amplitudes turn out to be dependent on the momentum operators. The transformed tunnelling Hamiltonian reads

$$\sum_q \left[ t_S e^{\sum_n \varphi_n(z_0) \left( \frac{\hat{X}_n}{\lambda} + i\Im \frac{\hat{P}_n}{2M\hbar\omega_n^2} \right)} c^\dagger a_{q,S} + t_L e^{i\sum_n \varphi_n(z_0) \left( \Im \frac{\hat{P}_n}{2M\hbar\omega_n^2} \right)} c^\dagger a_{q,S} \right] + \text{H.c.}, \quad (6)$$

The physical analysis of Eq. (6) becomes more transparent after introducing the creation (annihilation) operators  $b_n^\dagger (b_n)$  for the elementary mechanical excitations (vibrons) of the  $n$ -th bending mode:  $\hat{X}_n = (b_n^\dagger + b_n) \chi_n / \sqrt{2}$ ,  $\hat{P}_n = i\hbar(\sqrt{2}\chi_n)^{-1} (b_n^\dagger - b_n)$ , where  $\chi_n / \sqrt{2} = \sqrt{\hbar / 2M\omega_n}$  is the position uncertainty in the vibrational ground state of the oscillator associated to the  $n$ -th mode. In this representation, the part of the Hamiltonian which describes the electron tunnelling processes between the STM tip and nanotube assume the form:

$$\sum_q t_S \left\{ e^{\sum_n (A_n^+ b_n^\dagger + A_n^- b_n)} c^\dagger a_{q,S} + e^{\sum_n (A_n^- b_n^\dagger + A_n^+ b_n)} a_{q,S}^\dagger c \right\}, \quad (7)$$

where the parameters  $A_n^\pm$ , which characterize the rates of the inelastic electronic transitions with absorption and emission of the vibronic quanta, are given by:

$$A_n^\pm = \frac{\varphi_n(z_0) \chi_n}{\sqrt{2}} \left( \frac{1}{\lambda} \mp \frac{\Im}{\hbar\omega_n} \right). \quad (8)$$

From the expressions of the parameters  $A_n^\pm$  shown in Eq. (8), it follows that the vibron emission processes are suppressed with respect to the absorption ones when electrons tunnel from the STM tip to the nanotube, whereas the absorption is suppressed and the emission promoted during the transitions in the opposite direction (which are described by the rightmost term of the transformed tunnelling Hamiltonian shown in Eq. (7)).

Moreover, one can achieve complete suppression of the vibron emission (absorption) for the given mode by varying the value of the electrostatic force  $\mathfrak{F}$ , which, according to Eq. (2), is controlled by the equilibrium distance between the STM tip and the nanotube,  $d$ . If after tunnelling from the STM tip, the electrons tunnel immediately off to the leads, the rate at which vibron emission processes occur will be substantially reduced and therefore, in the stationary regime, one can expect that the number of vibrons will be close to zero.

However, in order to drive a certain vibrational mode to its quantum ground state (which corresponds to an average number of vibrons much smaller than 1), its frequency must satisfy the condition  $\omega_n = \sqrt{2}\lambda\mathfrak{F}/\hbar$ . In the stationary regime, the root-mean-squared deviation of the center-of-mass position (which expresses the fluctuations of the mechanical state of the nanotube around its equilibrium configuration) is given by the square root of the following expression:

$$\langle(\hat{u}(0) - \langle\hat{u}^2(0)\rangle)^2\rangle = \chi_0^2 \sum_n \left(\frac{\omega_0}{\omega_n}\right) \varphi_n^2(0) \left(\langle b_n^\dagger b_n \rangle + \frac{1}{2}\right). \quad (9)$$

Taking into account that for the doubly clamped nanotube  $\omega_0/\omega_n \simeq (n+1)^{-2}$  and that  $\varphi_{2n+1}(0) = 0$  Eq. (9) indicates that the of center-of-mass fluctuations are mainly defined by the average number of the vibrons in the fundamental mode. Therefore, in order to suppress such fluctuations, this number should have the minimum possible value. On the basis of these considerations, in the rest of this paper we restrict our attention to the fluctuations of the fundamental bending mode, which will be considered as a quantum harmonic oscillator.

### 3. Quantum master equation

In order to perform a quantitative analysis of the phenomena discussed above, we start from the Lioville-von Neumann equation for the density matrix operator, which represents the state of the whole system

$$i\hbar \frac{d\rho}{dt} = [H, \rho(t)]. \quad (10)$$

In Born approximation with respect to the tunnelling amplitudes  $t_S, t_L$ , Eq. (10) can be recast in the following integral form:

$$\frac{d\rho}{dt} = \frac{1}{i\hbar} [H_0, \rho] - \frac{1}{\hbar^2} \int_{-\infty}^t dt' [H_T(t), [H_T(t'), \rho(t')]]. \quad (11)$$

where  $H_0 \equiv H_e + H_m + H_C$ , and  $\hat{\mathcal{A}}(t) = e^{iH_0 t/\hbar} \hat{\mathcal{A}} e^{-iH_0 t/\hbar}$ . Taking into account that the coupling between the nanotube and the electronic reservoirs (that is, the STM tip and the leads) is weak enough so that any back-action of the nanotube on their physical states is negligible, we can use the Ansatz:  $\rho(t) \approx \sigma(t) \otimes \rho_S \otimes \rho_L$ . The operator  $\sigma(t)$  is the *reduced* density matrix operator, which is defined as  $\sigma(t) \equiv \text{Tr}_{S+L}[\rho(t)]$  and represents the electronic and mechanical state of the oscillator. The density matrices  $\rho_\alpha$  describe the STM tip ( $\alpha = S$ ) and the leads ( $\alpha = L$ ) as electronic reservoirs at thermal equilibrium with temperatures  $T_S$  and  $T_L$ , which means that:

$$\text{Tr}(a_{q,\alpha}^\dagger a_{q,\alpha} \rho_\alpha) = (1 + \exp\{(E_{q,\alpha} - \mu_\alpha)/k_B T_\alpha\})^{-1} \equiv f_\alpha(E_{q,\alpha} - \mu)$$

where the  $\mu$  is the chemical potential of the STM tip and the leads, which is supposed to be the same.

In order to describe the nanotube dynamics it is convenient to project the reduced density matrix onto the subspaces corresponding to the electronic level in the nanotube being occupied or unoccupied. That amounts to multiply the operators  $c^\dagger c$  and  $c c^\dagger$  to Eq. (11) and trace over the electronic degrees of freedom of the nanotube. After this procedure, we obtain two coupled equations for the operators  $\sigma_0 \equiv \text{Tr}_e(\sigma c c^\dagger)$  and  $\sigma_1 \equiv \text{Tr}_e(\sigma c^\dagger c)$  which, in the high-temperature limit  $\hbar\omega/k_B T_L \ll 1$  turn out to be local in time [12]. For small displacements of the nanotube around the equilibrium configuration, the tunnelling amplitude  $t_S[\hat{X}]$  can be linearized, so that the equations of motion for  $\sigma_{0,1}$  have the form:

$$\begin{aligned} \frac{d\sigma_1}{dt} = & -\frac{i}{\hbar}[H_m, \sigma_1] + \frac{i\Im}{\hbar}[\hat{X}, \sigma_1] - \Gamma_L^- \sigma_1 - \Gamma_S^- \left( \sigma_1 + \frac{1}{\lambda} \{\sigma_1, \hat{X}\} + \frac{1}{\lambda^2} \{\sigma_1, \hat{X}^2\} \right) + \\ & + \Gamma_S^+ \left( \sigma_0 + \frac{1}{\lambda} \{\sigma_0, \hat{X}\} + \frac{1}{2\lambda^2} \{\sigma_0, \hat{X}^2\} + \frac{1}{\lambda^2} \hat{X} \sigma_0 \hat{X} \right) + \Gamma_L^+ \sigma_0 + \mathcal{L}_\gamma \sigma_1 \end{aligned} \quad (12a)$$

$$\begin{aligned} \frac{d\sigma_0}{dt} = & -\frac{i}{\hbar}[H_m, \sigma_0] + \Gamma_L^- \sigma_1 + \Gamma_S^- \left( \sigma_1 + \frac{1}{\lambda} \{\sigma_1, \hat{X}\} + \frac{1}{2\lambda^2} \{\sigma_1, \hat{X}^2\} + \frac{1}{\lambda^2} \hat{X} \sigma_1 \hat{X} \right) - \\ & - \Gamma_S^+ \left( \sigma_0 + \frac{1}{\lambda} \{\sigma_0, \hat{X}\} + \frac{1}{\lambda^2} \{\sigma_0, \hat{X}^2\} \right) - \Gamma_L^+ \sigma_0 + \mathcal{L}_\gamma \sigma_0, \end{aligned} \quad (12b)$$

where  $\Gamma_\alpha^+ \equiv \Gamma_\alpha f_\alpha(E_0)$ ,  $\Gamma_\alpha^- \equiv \Gamma_\alpha [1 - f_\alpha(E_0)]$  and  $\Gamma_\alpha \equiv 2\pi\hbar^{-1}|t_\alpha|^2\nu_\alpha$ ,  $\nu_\alpha$  being the density of states at the Fermi energy in the STM tip ( $\alpha = S$ ) and in the leads ( $\alpha = L$ ).

The operator  $\mathcal{L}_\gamma$  in Eqs. (12a) and (12b) models the relaxation of the oscillator towards thermal equilibrium with the phononic bath in the leads, a process characterized by the rate  $\gamma \equiv \omega_0/Q$ , where  $Q$  is the quality factor. On the basis of general considerations regarding quantum dissipative systems [13],  $\mathcal{L}_\gamma$  can be explicitly written as:

$$\mathcal{L}_\gamma[\sigma] \equiv -\frac{i\gamma}{2\hbar}[\hat{X}, \{\hat{P}, \sigma\}] - \frac{\gamma}{2\chi_0^2} \coth(\hbar\omega_0/2k_B T_L)[\hat{X}, [\hat{X}, \sigma]]. \quad (13)$$

The mechanical state of the suspended nanotube can be characterized through the operator  $\sigma_+ \equiv \sigma_0 + \sigma_1$ , whose evolution in time is fully determined by Eq. (12a) and (12b), once that the operator  $\sigma_- \equiv \sigma_0 - \sigma_1$  is introduced. Furthermore, in order



to describe the stationary state of the oscillator, it is convenient to introduce the dimensionless operators  $\hat{x} = \hat{X}/\chi_0$ ,  $\hat{p} = \chi_0\hat{P}/\hbar$  and express the operators  $\sigma_{\pm}$  in the “Wigner function representation” [14], which is defined as:

$$W_{\pm}(x, p, t) = \int_{-\infty}^{+\infty} \frac{d\xi}{\pi} e^{-i2p\xi} \langle x - \xi | \sigma_{\pm}(t) | x + \xi \rangle \quad (14)$$

From Eqs. (12a) and (12b), it follows that the Wigner functions corresponding to the stationary solutions of the quantum master equations for the operators  $\sigma_+$ ,  $\sigma_-$  satisfy the equations:

$$\begin{aligned} (p\partial_x - x\partial_p)\overline{W}_+ &= \frac{\varepsilon_p}{2} \frac{\Delta\Gamma_{\Sigma}}{2\Gamma_{\Sigma}} \partial_p \overline{W}_+ + \frac{\varepsilon_p}{2} \partial_p \overline{W}_- + \varepsilon_p^2 \frac{\Gamma_S}{2} \partial_p^2 \overline{W}_+ + \varepsilon_t^2 \frac{\Delta\Gamma_S}{4} \partial_p^2 \overline{W}_- + \\ &+ \frac{1}{Q} \partial_p(p\overline{W}_+) + \frac{\coth(\hbar\omega_0/2k_B T_L)}{2Q} \partial_p^2 \overline{W}_+ + \mathcal{O}(Q^{-2}, \varepsilon_t^4) \overline{W}_- \end{aligned} \quad (15a)$$

$$\begin{aligned} (p\partial_x - x\partial_p)\overline{W}_- &= \varepsilon_p \frac{\Delta\Gamma_{\Sigma}}{2\Gamma_{\Sigma}} \partial_p \overline{W}_- + \frac{\varepsilon_p}{2} \partial_p \overline{W}_+ - (\Delta\Gamma_{\Sigma} + 2\varepsilon_t \Delta\Gamma_S x) \overline{W}_+ - \\ &- (\Gamma_{\Sigma} + 2\varepsilon_t \Gamma_S x) \overline{W}_- + \mathcal{O}(Q^{-1}, \varepsilon_t^2) \overline{W}_- \end{aligned} \quad (15b)$$

where  $\varepsilon_t = \varphi_0(z_0)\chi_0/\lambda$ ,  $\varepsilon_p = \varphi_0(z_0)\Im/\chi_0 M\omega_0^2$ , while the parameters related to the tunnelling processes have been rescaled in units of  $\omega_0$ , that is  $\Delta\Gamma_{\alpha} \equiv (\Gamma_{\alpha}^+ - \Gamma_{\alpha}^-)/\omega_0$ , where  $\alpha = S, L$ ,  $\Delta\Gamma_{\Sigma} = (\Delta\Gamma_S + \Delta\Gamma_L)/\omega_0$  and  $\Gamma_{\Sigma} = (\Gamma_S + \Gamma_L)/\omega_0$ .

Eqs. (15a) and (15b) can be solved by means of a perturbative expansion in the small coupling constants  $\varepsilon_p \sim \varepsilon_t \ll 1$  and inverse quality factor  $Q^{-1} \simeq \varepsilon_p^2, \varepsilon_t^2$ . At the zero-th order in the small parameters, the solution of Eqs. (15a) and (15b), has the form  $\overline{W}_+^{(0)} = w(A)$ ,  $\overline{W}_-^{(0)} = -(\Delta\Gamma_{\Sigma}/\Gamma_{\Sigma})w(A)$ , where  $A \equiv \sqrt{x^2 + p^2}$  and  $w$  is an arbitrary function.

The necessary and sufficient condition for  $w(A)$  to be a good zero-order approximation of  $\overline{W}_+$  is that any deviation from  $w(A)$  is at most of order  $(\varepsilon_t^2, \varepsilon_p^2, Q^{-1})$  in comparison to  $w(A)$ . Then, by replacing the expressions of  $\overline{W}_{\pm}$  up to the second-order corrections in the small parameter into Eqs. (15a) and (15b), and neglecting all the contributions except the zero-th order ones, we obtain an equation that determines the form of  $w(A)$ .

The first order corrections to the function  $\overline{W}_+^{(0)} = w(A)$  vanish after performing the transformation  $x \rightarrow x - \bar{x}$  (where  $\bar{x} = (\varepsilon_p/2)(1 - \Delta\Gamma_{\Sigma}/\Gamma_{\Sigma})$ ), which can be considered as a shift of the reference frame that accounts for the nanotube deformation induced by the average electrostatic force.

## 4. Results

Following the perturbative procedure described above, it turns out that, for the function  $w(A)$  to be an appropriate approximation for  $\overline{W}_+$ , it must satisfy the following first-

order linear differential equation:

$$\left( \frac{\varepsilon_p^2 \Gamma_\Sigma (1 - (\Delta\Gamma_\Sigma/\Gamma_\Sigma)^2)}{4} + \frac{\varepsilon_p^2 \Gamma_S \Gamma_\Sigma - \Delta\Gamma_S \Delta\Gamma_\Sigma}{4 \Gamma_\Sigma} + \frac{1}{2Q} \coth \left( \frac{\hbar\omega_0}{2k_B T_L} \right) \right) \partial_A w = -2 \left( \frac{\varepsilon_p \varepsilon_t \Gamma_L \Delta\Gamma_S - \Gamma_S \Delta\Gamma_L}{2 \Gamma_\Sigma (1 + \Gamma_\Sigma^2)} + \frac{1}{Q} \right) Aw \quad (16)$$

Now we focus our attention on the regime in which the transport of charge from the STM tip to the leads is activated by the gradient of temperature between the tip and the leads. In order to have an appreciable rate of tunnelling between the STM tip and the nanotube, the STM tip temperature must satisfy  $k_B T_S \sim E_0$ , so that the electronic states at the energy of the electronic level of the nanotube have a good chance to be populated, that is  $f_S(E_0) \simeq 1/2$ . At the same time, the temperature in the leads should be much lower than  $T_S$ , so that the electronic states in the leads at the energy corresponding to the electronic level in the nanotube have a good chance to be empty, which means  $f_L(E_0) \simeq 0$ . Furthermore, the temperature of the leads should be high on the scale defined by the vibrational quantum energy, that is  $k_B T_L \gg \hbar\omega_0$ .

For what concerns the best cooling performance, we already observed that it is expected to be achieved if the electrons, after having tunneled to the nanotube from the STM tip, tunnel quickly to the leads rather than being transferred back to the STM tip. That corresponds to the situation in which  $\Gamma_S/\Gamma_L \ll 1$ . Under these conditions, the quasi-distribution function  $w(A)$  has the following Gaussian form:

$$w(A) = \frac{1}{\pi\theta^2} e^{-\frac{A^2}{\theta^2}}, \quad (17a)$$

$$\theta^{-2} = \frac{\frac{\varepsilon_p \varepsilon_t}{2(1+\Gamma_\Sigma^2)} + \frac{\Gamma_\Sigma}{\Gamma_S \Gamma_L} \frac{1}{Q}}{\frac{\varepsilon_p^2}{2(1+\Gamma_\Sigma^2)} + \frac{\varepsilon_t^2}{4} + \coth(\hbar\omega_0/2k_B T_L) \frac{\Gamma_\Sigma}{\Gamma_S \Gamma_L} \frac{1}{Q}}. \quad (17b)$$

We remark that, by virtue of the Gaussian form of the stationary state, the root-mean-square fluctuations of the nanotube center-of-mass position are just proportional to the width of the quasi-distribution function, i. e.  $\langle X^2 \rangle^{1/2} = \theta/\sqrt{2}$ .

For an oscillator coupled to an equilibrium environment, in the high-temperature limit,  $\theta^2$  becomes proportional to the temperature of the phononic bath in the leads,  $\theta^2 \sim T_L$  (in agreement with Einstein's relation), while it reduces to 1 for the  $w(A)$  that describes the quantum fluctuations of the oscillator in the ground state.

The denominator of the ratio that defines  $\theta^{-2}$  describes the diffusion in the energy space of the oscillator induced by the stochastic electronic tunnelling processes, whereas the numerator represents the effective damping generated by them (see Eq. (17b)).

In order to understand the physical origin of the stationary state of the oscillator determined by the temperature-activated electron tunnelling, let us consider the limit  $Q \rightarrow \infty$ , that is the situation in which the nanotube is decoupled from the equilibrium environment. In this limit the quasi-distribution function is determined only by the rates of the inelastic tunnelling transitions induced by the polaronic and tunnelling

electromechanical couplings, which are characterized by the parameters  $\varepsilon_p$  and  $\varepsilon_t$ , respectively. The stationary state in this limit is characterized by a width given by

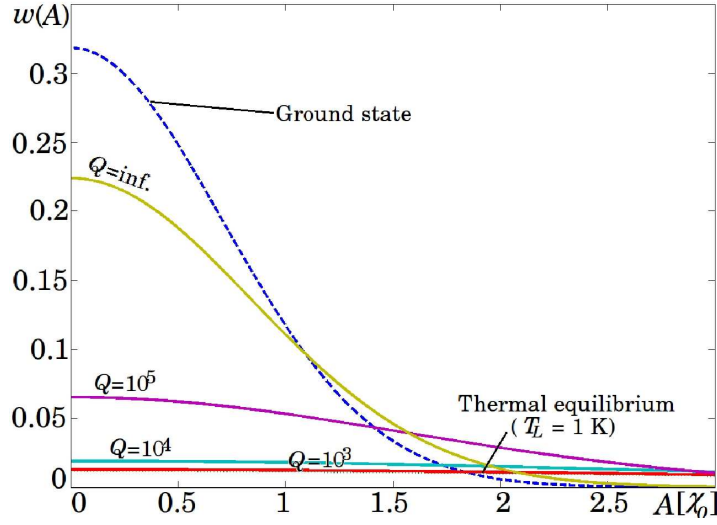
$$\theta_{Q \rightarrow \infty}^2 = \sqrt{\frac{1}{2}(1 + \Gamma_\Sigma^2)} \left( \eta + \frac{1}{\eta} \right), \quad (18)$$

where  $\eta = \sqrt{2\varepsilon_p^2/\varepsilon_t^2(1 + \Gamma_\Sigma^2)}$ . It follows from Eq. (18) that for both strong and weak polaronic coupling  $\theta$  is larger than 1, which corresponds to a state that is far from the quantum ground state. Nevertheless, in a suitable range of values of  $\varepsilon_p$ ,  $\theta^2$  is significantly smaller than  $\coth(\hbar\omega_0/2k_B T_L)$ , which means that the stationary state can be interpreted as a thermal state characterized by an effective temperature smaller than  $T_L$ . Correspondingly, the root-mean-square fluctuations of the nanotube center-of-mass position are smaller than the value determined in the thermal equilibrium case, therefore the mechanical subsystem is effectively *cooled*. The minimum value that  $\theta^2$  can reach as a result of the interaction of the nanotube with the nonequilibrium electronic environment is given by  $\theta_{min}^2 = \sqrt{2(1 + \Gamma_\Sigma^2)} > 1$ .

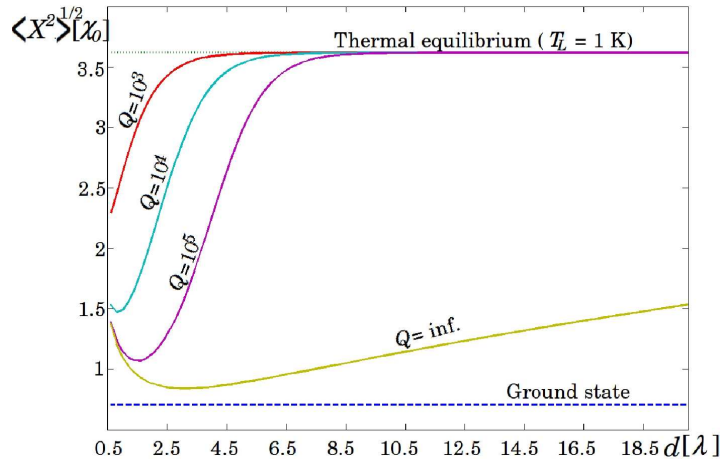
From Eq. (18), one can see that the effective cooling of the mechanical vibrations induced by the nonequilibrium environment requires the presence of both the mechanisms of electromechanical coupling in order to exist. In the case in which only one mechanism is active, it follows from Eq. (17b) that it can only generate diffusion in energy space, which results in a broadening of the quasi-distribution function. We stress that this behavior is characteristic of the nonequilibrium situation considered here, since in the case of coupling with an equilibrium environment, both damping and diffusion are present, independently of the type of interaction.

The stationary quasi-distribution function generated by the temperature drop from the STM tip to the leads is plotted with respect to the amplitude  $A$  (expressed in units of  $\chi_0$ ) in Fig. 2, for different values of the quality factor. Furthermore, we can compare the size of the root-mean-square fluctuations of the nanotube center-of-mass position in the ground state, the thermal equilibrium state and the electronically-induced stationary state as a function of the equilibrium distance between the nanotube and the STM tip,  $d$ , for different quality factors, as shown in Fig. 3. The curves shown in Figs. (2) and (3) indicates that the interaction between the electron tunnelling current and the oscillator can be interpreted as an effective cooling of the mechanical degrees of freedom.

In order to detect experimentally the cooling effect predicted above, the most direct approach consists in the measurement of the root-mean-square fluctuations of the nanotube center-of-mass position. Regarding this point, it has been argued since a long time ago that the STM (combined with a current amplifier) can provide the basic building block for a quantum-limited position displacement sensor. The tunnelling current that can be measured at the output of such a device contains information about the displacement of the mechanical system under investigation but, at the same time, perturbs it with a very small back-action force, being this mainly due to the random momentum transfer associated with the tunnelling electrons [15].



**Figure 2.** Comparison between the Wigner functions corresponding to the ground state of the oscillator, the thermal equilibrium state at temperature  $T_L$  and the stationary state induced by the nonequilibrium electronic environment for different quality factors. Values of the relevant parameters:  $\omega_0 = 10^9$  Hz,  $\Gamma_S = 5 \cdot 10^6$  Hz,  $\Gamma_L = 10^8$  Hz,  $\varepsilon_t = 0.19$ ,  $\varepsilon_p = 0.14$ .



**Figure 3.** Comparison between the fluctuations of the root-mean-square fluctuations of the center-of-mass nanotube position  $\sqrt{\langle X^2 \rangle}$  calculated with the Wigner functions corresponding to the ground state of the oscillator, the thermal equilibrium state at temperature  $T_L$  and the stationary state induced by the nonequilibrium electronic environment as a function of the equilibrium distance between the STM tip and the nanotube,  $d$ , for different quality factors. Values of the relevant parameters:  $\omega = 10^9$  Hz,  $\Gamma_S = 10^8$  Hz,  $\Gamma_L = 10^8$  Hz,  $\lambda = 10^{-10}$  m,  $\varepsilon_t = 0.19$ .

In conclusion, we have studied the coupled dynamics of the mechanical and electronic degrees of freedom of a suspended-nanowire-based NEMS wherein the movable element is in tunnelling contact with the tip of an STM and two supporting metallic leads. Our analysis shows that, in the regime in which the electron transport is activated only by the temperature difference between different parts of the device, an effective cooling of the mechanical degrees of freedom can be achieved. This result depends crucially on the interplay between tunnelling and electrostatic coupling that characterizes the system considered here. The interaction of the suspended nanowire with the nonequilibrium environment provided by the tunnelling current reduces the amplitude of the root-mean-square fluctuations of the center-of-mass position of the nanowire. This effect could be in principle detected experimentally thanks to the almost-quantum limited sensitivity of the STM as a displacement sensor.

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